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Seismic response of non-symmetric structures using the 1990 NBCC

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ABSTRACT

The seismic response of three non-symmetric structures is evaluated using a 3-D dynamic spectral analysis. The analyses are calibrated to the static base shear obtained using the 1990 NBCC. For the spectral analysis, three dynamic degrees of freedom are used at the centre of mass of each floor. In applying the NBCC, four techniques are proposed for evaluation of the centre of rigidity at each floor and corresponding torsional moments. Results show, for a given global base shear, different distributions between the various lateral load-resisting elements for the various techniques.

INTRODUCTION

When the centre of rigidity (CR) of a structure is not coincident with its centre of mass (CM), torsional effects are introduced. If a 3-D dynamic analysis is performed, with 3 degrees of freedom at the CM of each floor level, there is no need to evaluate the structure's CR. However, the obtained global base shear must be calibrated to the 1990 NBCC static base shear. When the 1990 NBCC is used for non-symmetric structures, the torsional moments at the CR are obtained by multiplying the forces at each floor level by the design eccentricity e_x where

$$e_x = 1.5e + .1D \tag{1}$$

$$e_x = .5e + .1D \tag{2}$$

The value of e is simply the distance between the CM and CR and the second term accounts for the accidental eccentricity which represents 10% of the width of the structure perpendicular to the earthquake direction. Four methods are used to evaluate the position of the CR.

Relative rigidities (Method A)

The most common method, proposed by Blume et al (1961), defines the X and Y coordinates of the CR, at level r, by

$$X_{CR} = \sum_{i=1}^{n} X_i \, k_{yi} \; / \; \sum_{i=1}^{n} k_{yi} \tag{3}$$

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$$Y_{CR} = \sum_{i=1}^{n} Y_i \, k_{xi} \Big/ \sum_{i=1}^{n} k_{xi} \tag{4}$$

k_{xi} and k_{yi} is the stiffness of the lateral load resisting element i at X_i and Y_i and n is the total number of such elements.

Force and moment applied at each level (Method B)

It uses the same principle as Method A and was proposed by Cadotte (1990). It consists of applying systematically on a 3-D model, arbitrary forces F_x and F_y in both X and Y directions as well as a moment M_z at the CM of each floor level. The eccentricity is then given from the corresponding diaphragm rotations obtained at each level by:

$$e_x = \Theta_{Fx} \, M_z \, / \, \Theta_{Mz} \, F_x \tag{5}$$

$$e_y = \Theta_{Fy} \, M_z \, / \, \Theta_{Mz} \, F_y \tag{6}$$

Positions for overall zero rotations (Method C)

This method, proposed by Stafford-Smith and Vezina (1985) and by Cheung and Tso (1986), requires that for all positions of the CR at all levels of the buildings, a pure translation occurs with no torsional rotation. This method can be applied using 2D models for the lateral load-resisting elements connected by rigid links. From the applied lateral loads P_x and P_y, the shears Q_x or Q_y are evaluated, for element i, in both X and Y directions and at a given level r. The CR for given lateral loads is simply:

$$X_{CR,r} = \sum_{i=1}^{n} (Q_{yi,r} - Q_{yi,r+1}) \, X_i \, / \, P_{y,r} \tag{7}$$

$$Y_{CR,r} = \sum_{i=1}^{n} (Q_{xi,r} - Q_{xi,r+1}) \, Y_i \, / \, P_{x,r} \tag{8}$$

Positions for overall zero rotations in 3-D (Method D)

This method is identical to the previous one except that the modelling of the lateral load-resisting elements is performed is 3-D. Therefore the displacements perpendicular to the direction of load application and the rotations at floor levels are prevented.

NUMERICAL EVALUATION OF CENTRE OF RIGIDITIES

The four methods described above are evaluated numerically for three different structures that have a torsional eccentricity in one of the two directions.

Building 1 is a 25-storey structure with an important vertical set-back at about one-third of its height (88,8m). Four lateral load-resisting elements are present in the Y-direction while three elements are placed symmetrically in the X-direction as shown in Fig. 1a. In order to simplify the model, floors have been lumped together. Fig. 1b shows the position of the CM vertically as well the CR using the four methods described earlier. It should be noted that methods C and D yield identical results and the value of CR can undergo wild excursions outside the building enveloppe.

Building 2 is a 27-storey structure shown in Fig. 2a with a height of 97,2m. The two cores provide the major lateral load-resisting elements. Core #1 is continuous throughout the height of the building while core #2 stops half-way. Fig. 2b shows the position of the CR. Once again, Method D shows erratic values but nonetheless consistent with the definition for the CR. Method C has been omitted as this structure cannot be appropriately analyzed in 2D due to the core/frame interactions in 3D.

Building 3 is a parking structure with eleven stories and a height of 27,5m. The lateral load-resisting elements consists of 3 cores and bracings have been added at the perimeter of the structure as shown in Fig. 3a. This structure has a double eccentricity but only the earthquake in the Y direction has been considered here. The other direction X has been analysed by Cadotte (1990). Due to the difficulty in evaluating the stiffness of the peripheral frames which have a flexural as well as a shear contribution, Method A has been discarded for evaluation on the CR. Fig. 3b shows the CR for the three other techniques. Even though the CR for methods C and D are within the structure their vertical positions vary quite significantly.

EVALUATION OF TORSIONAL RESPONSE

The NBCC requires the evaluation of the torsional moments by multiplying the storey force by the design eccentricities evaluated using equations (1) and (2). Considering the very wild differences in the position of the CR using different techniques, it is essential for a designer to appreciate the impact of the torsional moments and the corresponding internal distributions on the lateral load resisting elements. Furthermore, some techniques concerning the evaluation of the CR do create excessively large torsional moments of opposite signs (fortunately) but leaves the designer wondering about their validity. Furthermore, the NBCC (Clause 4.1.9.1(24)) states explicitly that when the CR and CM of the different floors do not lie approximately on a vertical line, a dynamic analysis shall be carried out to determine the torsional response. However, the base shear from a dynamic analysis must be calibrated to the static base shear. When a 3-D multi-modal spectral analysis is performed where modes are combined, such a calibration is not quite obvious.

Calibration of a dynamic analysis

The procedure is presented in point-form and is summarized below:

1) A 3-D model using three dynamic degrees of freedom at the CM of each floor is created. No allowance is made for the accidental eccentricity of the mass but it could easily be accounted for.

2) The periods, mode shapes, for at least the first ten modes and the corresponding generalized masses M^* and participation factors Γ_x, Γ_y and Γ_θ for each mode are obtained.

3) For a given response spectrum, using an annual probability of 0,0021, and earthquake direction for the site considered, the spectral modal accelerations S_a are evaluated. No allowance is made for the ductility by introducing reduced spectra.

4) For an earthquake in X direction, the base shears and torsions are evaluated for each mode i.

$$V_{xi} = M_i^* \, \Gamma_{xi}^2 \, S_{ai} \tag{9}$$

$$V_{yi} = M_i^* \, \Gamma_{xi} \, \Gamma_{yi} \, S_{ai} \tag{10}$$

$$T_{\theta i} = M_i^* \, \Gamma_{xi} \, \Gamma_{\theta i} \, S_{ai} \tag{11}$$

Similar expressions are obtained for an earthquake in a different direction. The torsional spectrum has not been considered in this study but could be introduced (Awad and Humar, 1984).

5) For each earthquake direction, the modal base shears are combined using either RSS or CQC techniques. This base shear corresponds to the translational excitation. For example, for an earthquake in X

$$V_{DYN} = (\sum_{i=1}^{n} V_{xi}^2)^{1/2} = (\sum_{i=1}^{n} M_i^* \Gamma_{xi}^2 S_{ai})^{1/2} \tag{12}$$

6) Using the 1990 NBCC, the base shear formula is given by:

$$V_{90} = .6(vSIFW)/R \tag{13}$$

a calibration factor α is obtained where

$$\alpha = V_{90} / V_{DYN} \tag{14}$$

7) Finally for each lateral load resisting element, using the combined modal spectral response, the results of the dynamic analysis are multiplied by α

It is obvious that the dynamic base shear, for a given direction, must be evaluated using Eq. (12). If a software does not provide such values or the required information to calculate V_{DYN} manually, the combined modal results for the individual elements cannot be used to evaluate the dynamic base shear due to loss of signs in the modal combinations.

Table 1 shows the results of the base shears using the above procedure assuming the buildings exist in Montreal and the force reduction factor R=1. Similarly I and F were set to unity. For the spectral analysis using 5% damping, the ground motion was assumed to have a PHA=.18g and a PHV=.097m/s. The values for α vary from .90 to 1.31 depending on the building configuration, torsional coupling as well as the contribution of higher modes.

DISCUSSION OF RESULTS

Table 2 shows the resultant torsional moment M_1 and M_2, obtained using Eqs. (1) and (2), for the various methods of evaluating the eccentricity. Only the effect of the governing torsional moment and the shear, at each floor level, is shown in the figs. 1c, 1d, 1e, 2c, 2d, 3c and 3d. Other details are given by Cadotte (1990).

For Building 1, the static and dynamic analysis shows large differences in the torsional response. In Figs. 1c and 1d, the shears in elements 1 and 2 are overestimated by a factor of nearly 2 while element 3, shown in Fig. 1e and is furthest from the CM, has its base shear underestimated by about the same factor.

For Building 2, the shears in cores 1 and 2 are close at the base as shown in Figs. 2c and 2d. Where the largest difference occurs is at level 15 where core No. 2 is discontinued. The dynamic analysis shows a smoother transition while all the static methods yield a much larger discontinuity.

For Building 3, the shears in elements 9 and 11 are shown in Figs. 3c and 3d. The results are surprisingly similar at the base since little difference is shown in the torsional moments using methods B and C. Also, as shown in Fig. 3b the CM and CR are within the bounds of the elements considered.

CONCLUSIONS

For non-symmetric structures where the centre of rigidity and centre of mass do not lie approximately on a vertical line, a 3D dynamic analysis procedure is presented. This method has the advantage of avoiding the complex evaluation of CR of the structure and yields, for a calibrated base shear to the static base shear, a better internal distribution of the shears arising from the torsional response. The higher flexural or flexural-torsional modes are accounted for. The only missing item in this procedure is the evaluation of a torsional response spectrum.

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TABLE 1. Calibration factors for a 3D dynamic analysis

Building	Quake Direction	1990 NBCC		Dynamic		α
		T(s)	V_{90} (kN)	T(s)	V_{DYN} (KN)	
1	Y	2,50	25 300	3,06	19 300	1,31
2	X	2,53	15 900	2,38	17 740	0,90
3	X	0,95	21 900	1,25	22 960	0,95

TABLE 2. Resultant torsional moments at base using 1990 NBCC

Building	Method	M_1 (kN.m) eq.1	M_2 (kN.m) eq.2
1	A	-1 260 000	- 45 000
	B	- 711 000	94 000
	C	171 000	197 000
2	A	- 134 400	44 500
	B	- 79 100	63 000
	D	- 57 100	63 900
3	B	256 200	- 73 400
	C	227 700	- 37 500

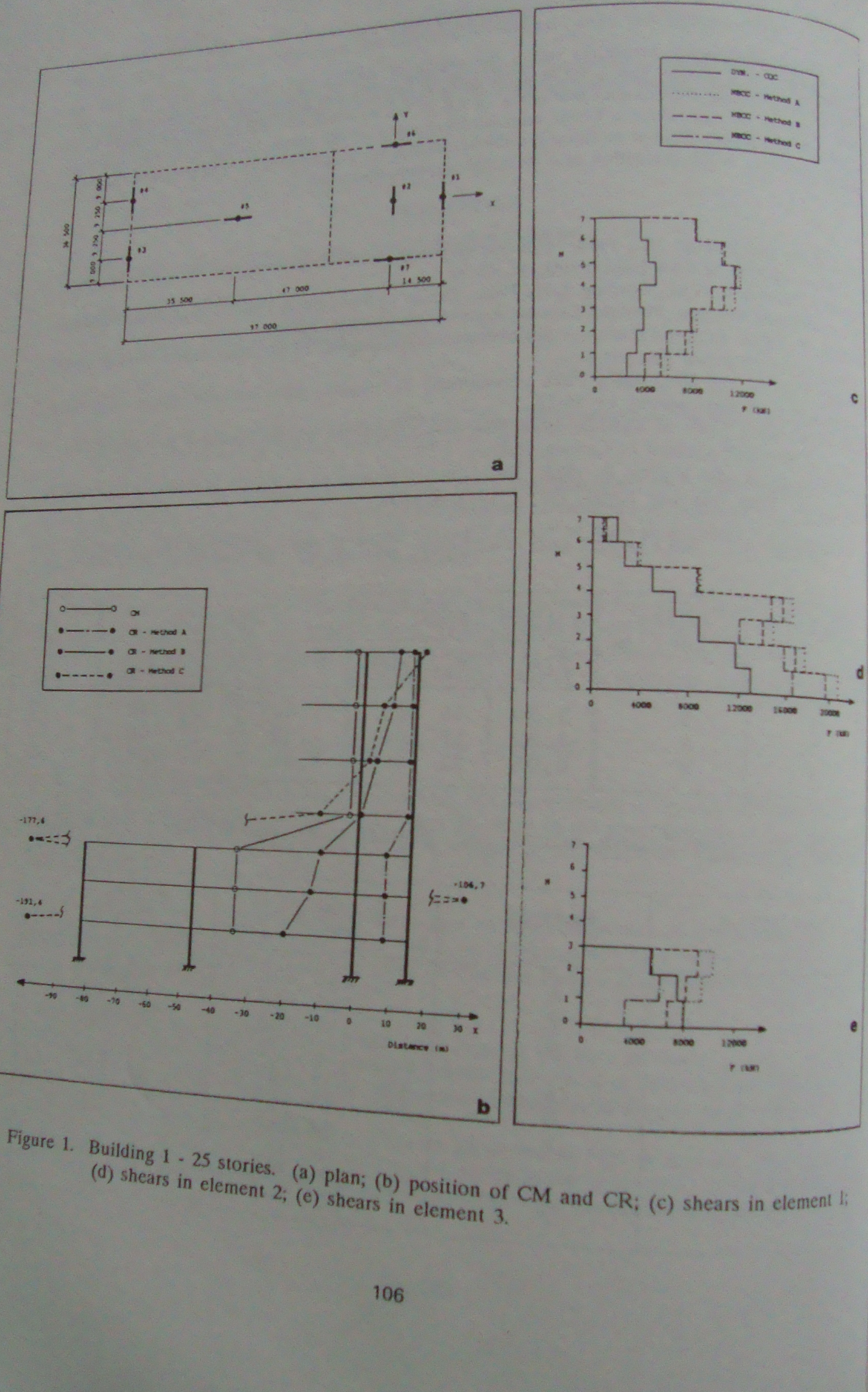

Figure 1. Building 1 - 25 stories. (a) plan; (b) position of CM and CR; (c) shears in element 1; (d) shears in element 2; (e) shears in element 3.

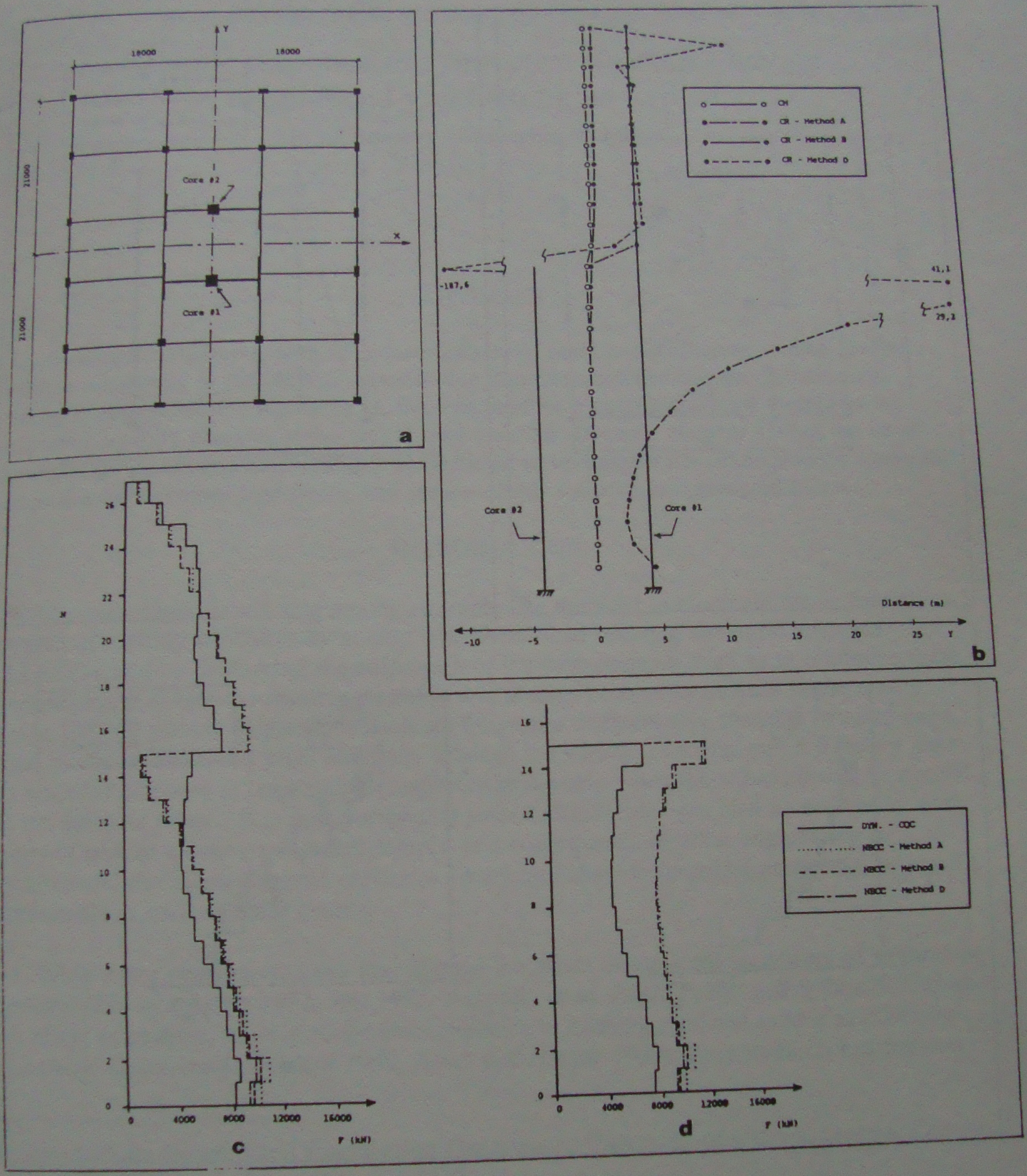

Figure 2. Building 2 - 27 stories. (a) plan; (b) position of CM and CR; (c) shears in core #1; (d) shears in core #2.

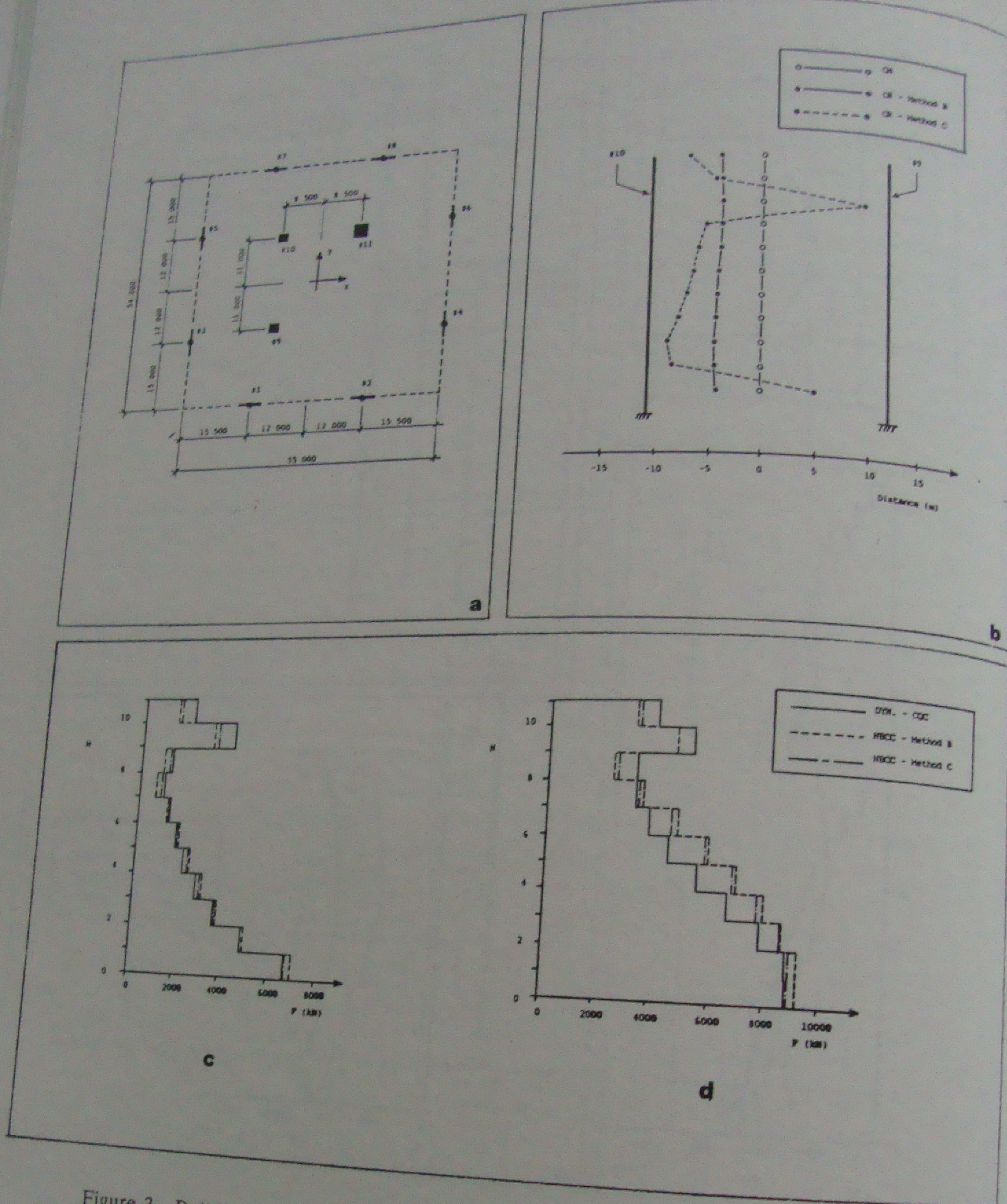

Figure 3. Building 3 - 11 stories. (a) plan; (b) position of CM and CR in Y-Z plane; (c) shears in element #9; (d) shears in element #11.